1. Let ABC be an isosceles triangle with $AB = AC$ and let Γ denote its circumcircle. A point D is on arc AB of Γ not containing C. A point E is on arc AC of Γ not containing B. If $AD = CE$ prove that BE is parallel to AD.

2. Find all triples (p, q, r) of primes such that $pq = r + 1$ and $2(p^2 + q^2) = r^2 + 1$.

3. A finite non-empty set of integers is called 3-*good* if the sum of its elements is divisible by 3. Find the number of non-empty 3-good subsets of $\{0, 1, 2, \ldots, 9\}$.

4. In a triangle ABC, points D and E are on segments BC and AC such that $BD = 3DC$ and $AE = 4EC$. Point P is on line ED such that D is the midpoint of segment EP. Lines AP and BC intersect at point S. Find the ratio BS/SD.

5. Let a_1, b_1, c_1 be natural numbers. We define

$$a_2 = gcd(b_1, c_1)\,, \quad b_2 = gcd(c_1, a_1)\,, \quad c_2 = gcd(a_1, b_1)\,,$$

and

$$a_3 = lcm(b_2, c_2)\,, \quad b_3 = lcm(c_2, a_2)\,, \quad c_3 = lcm(a_2, b_2)\,.$$

Show that $gcd(b_3, c_3) = a_2$.

6. Let $P(x) = x^3 + ax^2 + b$ and $Q(x) = x^3 + bx + a$, where a, b are non-zero real numbers. Suppose that the roots of the equation $P(x) = 0$ are the reciprocals of the roots of the equation $Q(x) = 0$. Prove that a and b are integers. Find the greatest common divisor of $P(2013! + 1)$ and $Q(2013! + 1)$.