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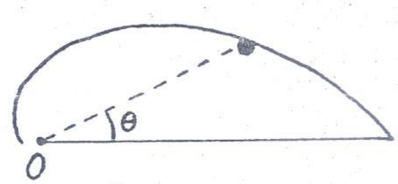
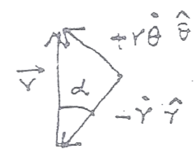
Q1.a A particle of mass m slides along a frictionless track, fixed in the horizontal plane, of the form $r = r_0 \exp(-a\theta)$ where a is a positive constant . Its initial speed at $t = 0$ is v_0 when $\theta = 0$. Recall that in polar coordinates $\vec{\boldsymbol{r}} = r\,\hat{\boldsymbol{r}}$, $\frac{d\vec{\boldsymbol{r}}}{dt} = \vec{\boldsymbol{v}} = \frac{dr}{dt}\hat{\boldsymbol{r}} + r\frac{d\theta}{dt}\,\hat{\theta}$, $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors in the radial direction and a direction perpendicular to $\hat{\boldsymbol{r}}$ respectively.



(i) Find the θ dependence of both the speed and the angle α that the velocity vector makes with the radial line connecting origin O with the particle. (2 + 2)

Only force acting on the particle is the Normal force N which does no work. (1)

Thus $v=v_0$ throughout. (1)



Let α be the angle made by the velocity vector with the radial direction. Then

$\tan\alpha = \frac{d\theta}{dt} r \big/ -\frac{dr}{dt}$ = 1/a and hence is a constant. (2)

$v(\vartheta)$ = constant = v_0 and independent of θ $\alpha(\vartheta)$ = constant = $\tan^{-1}1/a$ and independent of θ

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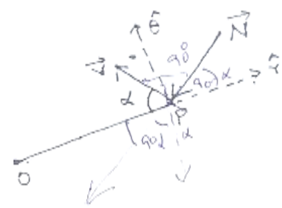
(ii) Obtain an expression for the rate of change of angular momentum $\frac{dL}{dt}$ about O in terms of α and other related quantities. (4)

Angular momentum about O : $L = mrv_0 sin\alpha$ directed out of the plane of motion



(1)

$\frac{dL}{dt} = m\frac{dr}{dt}\ v_0\ \sin\alpha\ =\ -mv_0^2\ sin\alpha\ cos\alpha$ (3)

dL/dt = $-mv_0^2\ sin\alpha\ cos\alpha$

(iii)) Obtain an expression for the normal force N exerted by the track on the particle in terms of θ , α and other related quantities. (2)

Torque about O: $\tau = -Nr\ cos\alpha$

Hence $N = \frac{mv_0^2}{r_{0\sqrt{1+a^2}}}\ e^{a\theta}$ (2)

N(θ) $= \frac{mv_0^2}{r_{0\sqrt{1+a^2}}}\ e^{a\theta}$

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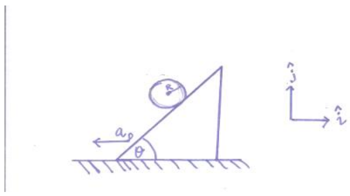
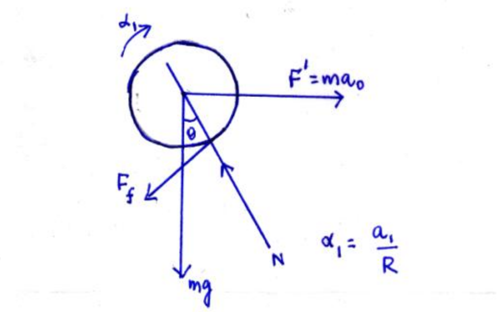
Q 1 b) A cylinder of mass M and radius R is placed on an inclined plane with angle of inclination θ. The inclined plane has acceleration a_0 with respect to an inertial frame as shown in the figure.



Assuming the cylinder rolls without slipping,

(i) Draw the free body diagram of the cylinder in the frame of the inclined plane. (2)



(ii) Find the acceleration $\vec{\boldsymbol{a}}$ of the center of mass of the cylinder with respect to an inertial frame. (8)

Choose the noninertial frame of the inclined plane. Let the acceleration of the cylinder in this frame be a_1 and the angular acceleration α_1.

$N - mg\,cos\theta - F'\,sin\theta = 0$ (1)

$-mgsin\theta + F'cos\theta - F_f \;= ma_1$ (1)

$F_f R = I\alpha_1 \Rightarrow F_f \;= \frac{M}{2} a_1$ (2)

Since $F' \;= ma_0$

$ma_0 cos\theta - mg\,sin\theta - \frac{M}{2} a_1 = Ma_1$

$\Rightarrow a_1 = \frac{2}{3}\,(a_0\,cos\theta - g\,sin\theta)$ (2)

Thus $\vec{a} = \overrightarrow{a_1} + \overrightarrow{a_0} = (a_1\,cos\theta - \,a_0)\hat{\imath} + \,a_1 sin\theta\,\hat{\jmath}$ (2)

$\vec{\boldsymbol{a}} = \,(a_1\,cos\theta - \,a_0)\hat{\imath} + \,a_1 sin\theta\,\hat{\jmath}$ where $a_1 = \frac{2}{3}\,(a_0\,cos\theta - g\,sin\theta)$

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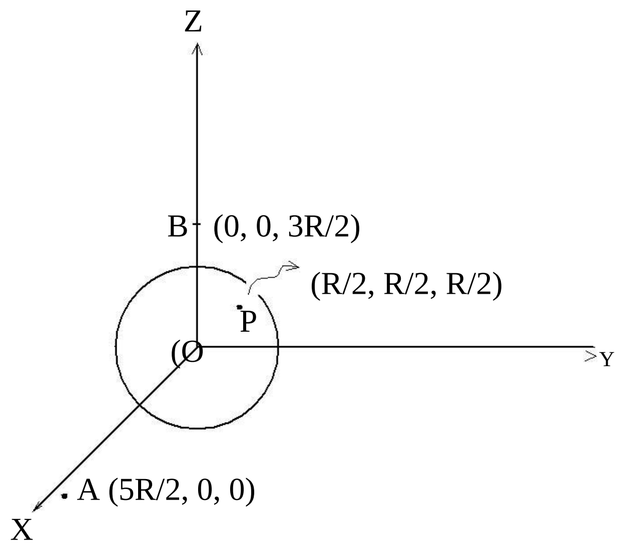
Q 2a) An uncharged solid spherical conductor of radius R has its centre at the origin of a cartesian frame OXY Z. Two point charges each of +Q are placed at fixed positions at (5R/2, 0, 0) and (0, 0, 3R/2). There will be induced charges on the sphere due to these charges.



(i) Find the electric field ***E*** at point P (R/2, R/2, R/2) due to the induced charges. (4)

The vector distance from A to P $\boldsymbol{r_{AP}}$ = -2R ***i*** + R/2 ***j*** + R/2 ***k*** (0.5)

The vector distance from B to P $\boldsymbol{r_{BP}}$ = R/2 ***i*** + R/2 ***j*** - R ***k*** (0.5)

Electric field at P due to charge at B $\boldsymbol{E_B} = Q/(4\pi\varepsilon_0)(\boldsymbol{r_{BP}}/|\boldsymbol{r_{BP}}|^3)$ (0.5)

Electric field at P due to charge at A $\boldsymbol{E_A} = Q/(4\pi\varepsilon_0)(\boldsymbol{r_{AP}}/|\boldsymbol{r_{AP}}|^3)$ (0.5)

Electric field at P due to charges at A and B $\boldsymbol{E_P} = \boldsymbol{E_A} + \boldsymbol{E_B}$ (0.5)

$\boldsymbol{E_P} = 2^{3/2}Q/(4\pi\varepsilon_0 R^2)(\,(9/2 - 2\text{X}3^{1/2})\boldsymbol{i} + (9/2 + 3^{1/2})\boldsymbol{j} + (9 - 3^{1/2})\boldsymbol{k}\,)$ (0.5)

Total field inside a conductor is 0 , thus induced field at P $\boldsymbol{E} = -\boldsymbol{E_P}$ (1.0)

$$\boldsymbol{E} = -\left(\frac{Q}{4\pi\varepsilon_0 R^2}\right)\frac{2\sqrt{2}}{27\sqrt{3}}\left(\left(-2\sqrt{3} + \frac{9}{2}\right)\boldsymbol{i} + \left(\frac{\sqrt{3}}{2} + \frac{9}{2}\right)\boldsymbol{j} + \left(\frac{\sqrt{3}}{2} - 9\right)\boldsymbol{k}\right)$$

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(ii) What is the electric potential V at point P due to the induced charges? (4)

The conductor is an equipotential surface. (0.5)
Thus the potential at any point is the same as the potential V(0.0,0) at (0,0,0)
$V(0,0,0) = Q/(4\pi\varepsilon_0 R)(2/3 + 2/5) = 16/15(Q/(4\pi\varepsilon_0 R))$ (1)
Potential at P $V(R/2, R/2, R/2) = 16/15(Q/(4\pi\varepsilon_0 R)) = V_A + V_B + V$ (0.5)
$V_A = Q/(4\pi\varepsilon_0 R)(2/9)^{1/2}$ (0.5)
$V_B = Q/(4\pi\varepsilon_0 R)(2/3)^{1/2}$ (0.5)
$V = Q/(4\pi\varepsilon_0 R)(16/15 - (2/9)^{1/2} - (2/3)^{1/2})$ (1.0)

$V = Q/(4\pi\varepsilon_0 R)(16/15 - (2/9)^{1/2} - (2/3)^{1/2})$

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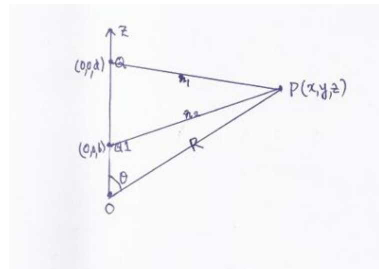
Q2b) (i) Consider two point charges Q and Q1 at the points (0,0,d) and (0,0,b) with d > b. Express Q1 and b in terms of Q, d and R such that the equipotential surface with 0 potential is a sphere of radius R for d>R. (2 + 2)



The potential at any point P(x,y,z) is

$V = 1/4\pi\varepsilon_0 \, (Q1/r_2 + Q/r_1)$

If the point P is to lie on the surface of a sphere of radius R with 0 potential, then

$Q1/(b^2+R^2-2bR\cos\theta)^{1/2} = -Q/(d^2+R^2-2Rd\cos\theta)^{1/2}$ (0.5)

This must be true for the points of the sphere on the z axis at θ = 0 and θ = π. (0.5)

At θ = 0

Q1/(b-R) = -Q/(d-R)

At θ = π.

Q1/(b+R) = -Q/(d+R)

i.e.Q1 d - Q1R = -Qb + QR Thus b>>R (0.5)

Q1d+Q1R = -Qb -QR

Which implies that Q1d =-2Qb

Q1 = -Qb/d

Further -2Q1R = -2Qr

Thus Q1 =- Q

Contradiction and hence b<R (0.5)

Thus at θ = 0

Q1/(R-b) = -Q/(d-R)

Thus Q1=-Q R/d (0.5)

Further Q1 = -Qb/R

Thus $b = R^2/d$ (0.5)

With this choice at any θ

$V = 1/4\pi\varepsilon_0 \, (-QR/d(R^4/d^2+R^2-2bR\cos\theta)^{1/2} + Q/(d^2+R^2-2Rd\cos\theta)^{1/2}) = 0$ (1.0)

> Q1= -QR/d
>
> $b = R^2/d$

(ii) Consider a point charge Q at (0,0,d) near a grounded spherical conductor of radius R < d. What is the electric field ***E*** on the charge Q due to the induced charges on the sphere? (2)

From the earlier problem, this problem is electrostatically identical with a charge Q at (0,0,d) and a charge -QR/d at $(0,0,R^2/d)$. Thus the electric field at Q is (1)

$\mathbf{E} = 1/4\pi\varepsilon_0 \, (QR/d \, (1/((d^2-R^2)/d)^2 \, (-\mathbf{z})$

$= 1/4\pi\varepsilon_0 \, (RdQ/(d^2-R^2)^2)(-\mathbf{z})$ (1)

> $\boldsymbol{E} = 1/4\pi\varepsilon_0 \, (RdQ/(d^2-R^2)^2)(-\mathbf{z})$

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2c)(i) Consider a point charge Q at (0,0,d) near a spherical conductor of radius $R < d$ charged to a potential V .Find the total charge q on the sphere. (1)

Using the superposition principle the problem is equivalent to Q at (0,0,d), $-QR/d$ at $(0,0,R^2/d)$ and $4\pi\varepsilon_0 VR$ at (0,0,0) . (0.5)

The total charge on the sphere is

$q = 4\pi\varepsilon_0 VR - QR/d$ (0.5)

$q = 4\pi\varepsilon_0 VR - QR/d$

(ii)Find the force ***F*** on the charge Q. (1)

The electric field at (0,0,d) is

$\mathbf{E} = 1/4\pi\varepsilon_0((RdQ/(d^2-R^2)^2(-\mathbf{z}) + 4\pi\varepsilon_0 VR/d^2(\mathbf{z}))$ (0.5)

Now $q = 4\pi\varepsilon_0 VR - QR/d$

Hence the force on Q is

$\mathbf{F} = Q/4\pi\varepsilon_0((q+QR/d)/d^2 - QR/d(d-R^2/d)^2)\mathbf{z}$ (0.5)

$\boldsymbol{F} = Q/4\pi\varepsilon_0((q+QR/d)/d^2 - QR/d(d-R^2/d)^2)\mathbf{z}$

(iii) If d>>>R find an expression for the force ***F*** on Q and also comment on its nature if $qQ > 0$ (1)

Expanding to the powers of R/d

$\mathbf{F} = qQ/4\pi\varepsilon_0 1/d^2\mathbf{z}$ **:** Nature repulsive (0.5 + 0.5)

$\boldsymbol{F} = qQ/4\pi\varepsilon_0 1/d^2\mathbf{z}$ Nature of force: repulsive

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(iv) If d=R + δ with δ→0 find an expression for the force ***F*** on Q and also comment on its nature if qQ>0. (1)

Expanding in powers of δ/R

F=$Q^2/4\pi\varepsilon_0 1/4\ \delta^2$ **(-z)**: Nature attractive (0.5) + (0.5)

F = $Q^2/4\pi\varepsilon_0 1/4\ \delta^2$ **(-z)**
Nature of force: attractive

(v) Briefly explain the observation on the nature of the forces in parts (v) and (vi) (2)

When d>>>R, the electric field at d looks like the Electric field due to a charge q. Thus the force is repulsive. When d=R+δ with δ --→ 0, the field at d due to the negative induced charge is larger than the uniform field due to the conductor being charged to potential V and hence the attraction.

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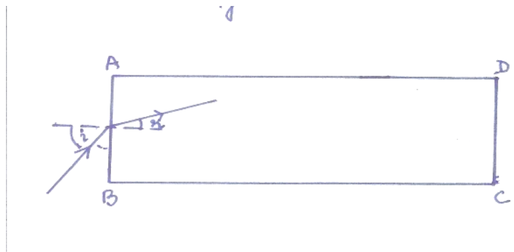
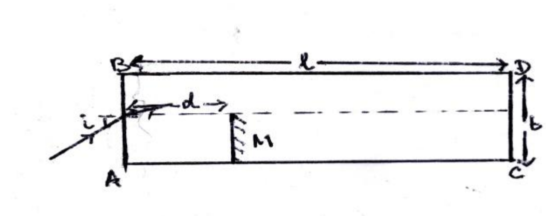
Q3 a). Light falls on the surface AB of a rectangular slab from air. Determine the smallest refractive index n that the material of the slab can have so that all incident light emerges from the opposite face CD. (2)

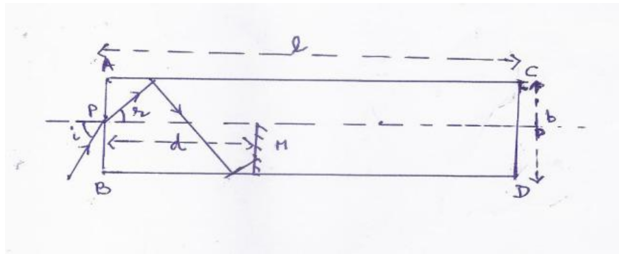


Let the light be incident at an angle i to the normal. If r is the angle of reflection then
Sin i = n Sin r, where n is the refractive index of the material.
If the ray is to be reflected from the surface perpendicular to AB
Sin (90 – r) = Cos r >= 1/n (1)
i.e. $(1 - \text{Sin}^2 r)^{1/2} >= 1/n$
i.e. $1 + \text{Sin}^2 i < = n^2$
The largest value of Sin2i = 1
Hence $n^2 >= 2$
$n = 2^{½}$ (1)

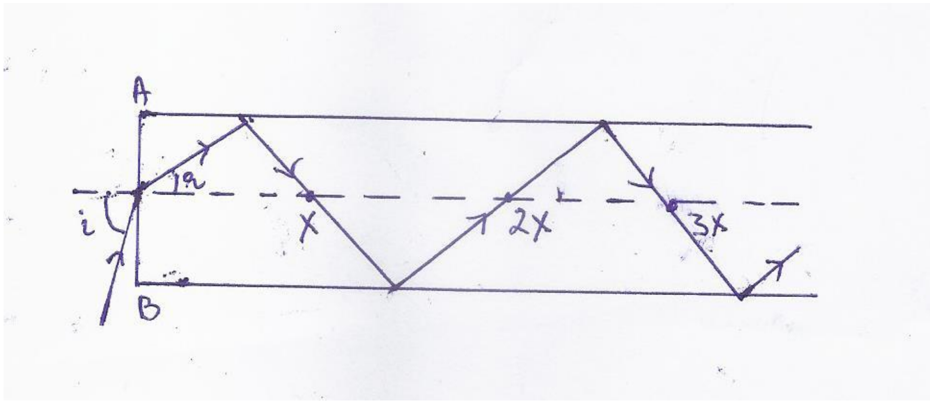
$n = 2^{1/2}$

3 b) (i) Consider a slab of transparent material of thickness b, length l and refractive index $n = 2^{1/2}$, surrounded by air. There is a mirror M of height b/2 inside the slab at a distance d from the face AB. The mirror is placed parallel to face AB, its lower end touching the face BC and the reflecting side towards AB. A narrow beam of monochromatic rays of light is incident at the center P of the face AB. The incident beam has an angle of incidence i lying between $-\pi/2$ and $\pi/2$. Ignoring diffraction effects find the values of i such that the rays emerge out of the opposite face CD. (6)





The critical angle for this material is $\pi/4$. If r is the angle of refraction at the face AB, then
$\sin r = \sin i / 2^{1/2} <= 1/2^{1/2}$
thus $r <= \pi/4$ (1)
thus the angle θ made by the face perpendicular to AB is such that
$\theta >= \pi/4$.
Hence all rays will either emerge from DC or get reflected from the mirror M and emerge from the face AB. (1)
To find the condition for reflection consider a ray incident at an angle i such that $0 <= i <= \pi/2$.
If the ray hits the central axis at a distance X after reflection then $X = b/\tan r$ where r is the angle of refraction.



It is clear from the figure that the ray with incident angle i will get reflected from the mirror if $X<d<2X$, or $3X < d < 4X$ or $5X < d < 6X$
i.e $(2m-1)X < d < 2m X$ where m =1, 2, 3,
or $(2m-1)\, b/\tan r < d < 2m\, b/\tan r$ (1)
Since tan r is positive
$(2m-1)\, b/d < \tan r < 2m\, b/d$
Now $i = \sin^{-1}(n \sin r) = \sin^{-1}(n \sin(\tan^{-1}(\tan r)))$

Since r and i are positively correlated , the condition for reflection can be written in terms of i by directly substituting values of the limiting value of tan r (1)

i.e. $\text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m-1)b/d)) < i < \text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}(2mb/d))$

Thus the condition for the rays to emerge from the opposite face CD is

$\text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m)b/d)) < i < \text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m+1)b/d))$ for m= 0, 1,2,3 etc. (1)

For the case when $-\pi/2 <= i <=0$ the condition gets reversed so that it will get reflected from the mirror for

2m X <= d <=(2m+1)X m=0,1,2.3

Thus the condition for emergence from face CD is

$\text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m-1)b/d)) < i < \text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m)b/d))$ for m= 1,2,3 etc. (1)

For $0 \leq i \leq \frac{\pi}{2}$

$\text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m)b/d)) < i < \text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m+1)b/d))$ for m= 0, 1,2,3 etc.

For $-\frac{\pi}{2} \leq i \leq 0$

$\text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m-1)b/d)) <- i < \text{Sin}^{-1}(n\ \text{Sin}(\tan^{-1}((2m)b/d))$ for m= 1,2,3 etc.

(ii) What happens if the angle of incidence $i = 0^0$? (2)

Practically there is no single ray. There is a bunch. Hence a part of the bunch shall be reflected and part transmitted through the opposite face.

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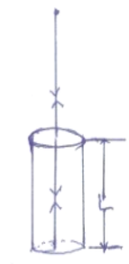
3 c) Whenever an object is heated the dimensions as well as the refractive index changes. Within a range of temperatures the changes are linear in the temperature differences. If the length and refractive index of a cylindrical object at room temperature (24^0C) is L and n respectively, then the changes ΔL and Δn are characterized by two properties of the body $\beta = \frac{1}{L}\frac{\Delta L}{\Delta T}$ and $\gamma = \frac{\Delta n}{\Delta T}$ where ΔT is the change in temperature. If β is known γ can be determined from observing the interference pattern due to reflection from the top surface and the bottom surface of a normally incident Laser beam of wavelength λ on a cylindrical sample of length L. As the temperature changes the fringe patterns shift.



(i) Obtain a relation between γ and the fringe shift m. (3)

Ans. Optical path difference is 2Ln. Constructive interference takes place if $2Ln = (m_1 + \frac{1}{2})\lambda$ (1)

When the Temperature changes from T to $T + \Delta T$ constructive interference takes place if

$2(n + \Delta n)(L + \Delta L) = (m_2 + \frac{1}{2})\lambda$ where $\Delta L = L\beta\Delta T$ (1)

Thus $2L\left(\frac{\Delta n}{\Delta T} + n\beta\right)\Delta T = (m_1 - m_2)\lambda = m\lambda$

$\gamma = \frac{\Delta n}{\Delta T} = \frac{m\lambda}{2L\Delta T} - n\beta$ (1)

$\gamma = \frac{m\lambda}{2L\Delta T} - n\beta$

(ii) In a real experiment with L=1.0cm., n= 1.515 and β = 7.19 X 10^{-16} $^0C^{-1}$ a Laser beam of λ = 632nm.was used. The data of the fringe shift with the temperature is given below:

m	1	2	3	4	5	6	7	8	9	10
T^0C	25.4	27.0	28.9	31.0	33.4	35.3	37.6	40.0	42.2	44.4

m	11	12	13	14	15	16	17	18	19	20
T^0C	46.6	48.9	51.6	54.0	56.2	58.6	61.2	64.0	66.4	69

Plot a graph between m and T^0C. (4)

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(Marking scheme: 0.5 for using the full graph paper; 0.5 for mentioning scaling of x axis; 0.5 for mentioning scaling of y axis; 2.0 for drawing a smooth graph and taking an average line; 0.5 points for choosing two far off points to calculate slope)

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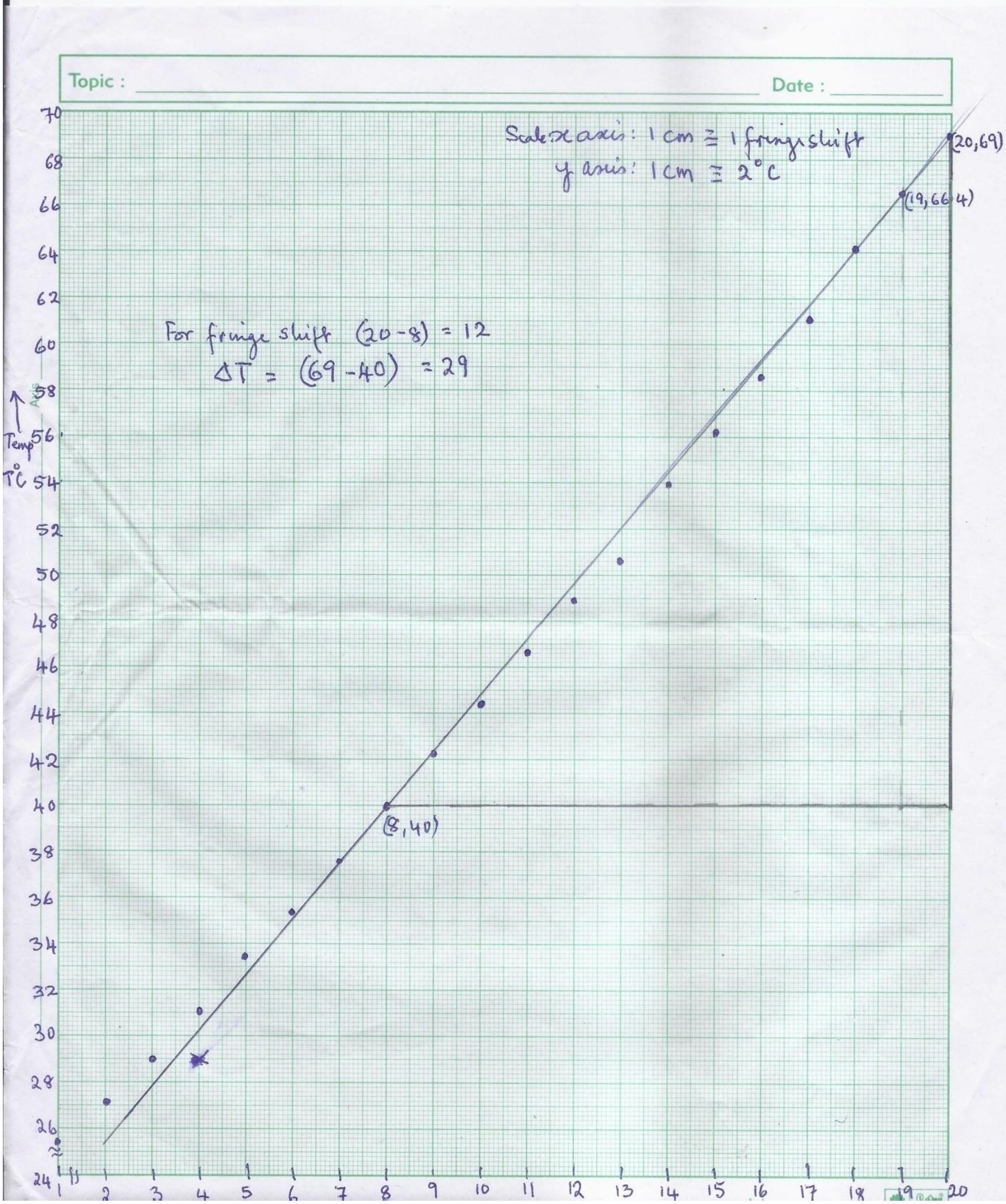

Topic :
Date :
Scale: x axis : 1 cm ≡ 1 fringe shift
y axis : 1 cm ≡ 2°C
For fringe shift (20-8) = 12
ΔT = (69-40) = 29
(20,69)
(19,66.4)
(8,40)
Temp T°C
70
68
66
64
62
60
58
56
54
52
50
48
46
44
42
40
38
36
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(iii) From the plot find γ. (3)

$\gamma = \frac{12 \times 632 \times 10^{-7}}{2 \times 1 \times 29} - 1.515 \times 7.19 \times 10^{-16}$

$= 13.075 \times 10^{-6}$ - 10.893×10^{-6} $=13.075 \times 10^{-6}$ (1)

$=1.30 \times 10^{-6}\ {}^{0}C^{-1}$ (1 for correct sig.figures 1 for units)

γ = $1.3 \times 10^{-6}\ {}^{0}C^{-1}$ (Remember the least count is till the first decimal digit)

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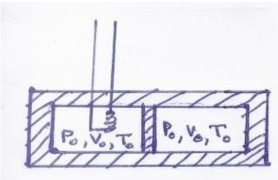
Q4) The figure shows a double chambered vessel with thermally insulated walls and partition. On each side there are n moles of an ideal monoatomic gas. Initially the pressure, volume and temperature on each side is P_0, V_0 and T_0 respectively. The heater in the first chamber supplies heat very slowly till the gas in the first chamber expands such that the pressure, volume and temperature of the gas on the left side is P_1, V_1, T_1 respectively. The pressure, volume and temperature of the gas in the right chamber is $P_2 = 27P_0/8$, V_2 and T_2 respectively.



a) Complete the Table below: (5)

$P_1 = 27P_0/8$	$V_1 = (2 - (8/27)^{3/5})V_0$	$T_1 = (27/4-(27/8)^{2/5})T_0$
$P_2 = 27P_0/8$	$V_2 = (8/27)^{3/5}V_0$	$T_2 = (27/8)^{2/5}T_0$

Since the gas is monoatomic $\gamma = 5/3$. Given $P_2 = 27P_0/8$. Since the system is in equilibrium $P_1 = 27P_0/8$(1)

Since chamber two is compressed adiabatically, $P_0V_0^{\gamma} = P_2V_2^{\gamma}$.

Hence $V_2 = (8/27)^{3/5} V_0$ (1)

$V_1 = 2V_0 - V_2 = (2 - (8/27)^{3/5})V_0$ (1)

$P_0V_0 = nRT_0$ and $P_2V_2 = nRT_2$ Hence $T_2 = (27/8)^{2/5}T_0$ (1)

$P_1V_1 = nRT_1$ Hence $T_1 = (27/4-(27/8)^{2/5})T_0$ (1)

b) Find the work ΔW done on the gas in the second chamber in terms of the molar specific heat and T_0.

(2)

Since the change is adiabatic $\Delta Q = 0$. Thus $\Delta W = -\Delta U$ (1)

$\Delta W = \int_{V_0}^{V_2} P\, dV = K \int_{V_0}^{V_2} \frac{dV}{V^\gamma} = \frac{K}{1-\gamma}\left[V_2^{1-\gamma} - V_0^{1-\gamma}\right] = \frac{1}{1-\gamma}[P_2V_2 - P_0V_0]$

With $\gamma = \frac{C_P}{C_v}$ and $C_P - C_v = R, \Delta W = -nC_v((27/8)^{2/5} - 1)T_0$ (1)

Alternatively $\Delta W = -\Delta U = -nC_v(T_2 - T_1) = -nC_v((27/8)^{2/5} - 1)T_0$ (1 + 1)

$\Delta W = nC_v((27/8)^{2/5} - 1)T_0$

c) Find the amount of heat ΔQ that flows into the first chamber in terms of the molar specific heat and T_0.

(3)

$\Delta Q = \Delta U + \Delta W = nC_v(T_1 - T_0) + nC_v((27/8)^{2/5} - 1)T_0$ (1) + (1)

$=> \Delta Q = \frac{19}{4} nC_vT_0$ (1)

$\Delta Q = \frac{19}{4} nC_vT_0$

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Q5 While performing an experiment with a liquid z (specific heat capacity 2.19kJ Kg K^{-1}), an experimenter starts with 200 ml of the liquid z in a beaker on a hot plate which is attached with scales to measure mass on it along with time. There is no lid over the beaker and the liquid is kept exposed to the surrounding. The experimenter inserts a thermometer such that it is always in contact with the liquid near the bottom of the beaker. The experimenter turns on the hot plate at t=0 min. and records the liquid temperature as well as the combined mass of the liquid, beaker and thermometer every minute. After 24 mins. he observes that there is no more liquid in the beaker. The observations are tabulated below:

Time (min)	0	1	2	3	4	5	6	7	8	9	10	11	12
Temp. (^{0}C)	24	25	28	37	50	64	77	90	102	113	124	134	143
Mass (gm)	310	310	310	310	310	310	310	310	310	310	310	307	307

Time (min)	13	14	15	16	17	18	19	20	21	22	23	24
Temp. (^{0}C)	152	158	160	160	160	159	160	160	161	160	161	-
Mass (gm)	305	302	288	264	241	214	190	165	138	110	79	69

Based on the above data

a) Plot a qualitative graph between the rate of change of temperature vs. temperature i.e. ΔT/Δt vs T. (5)

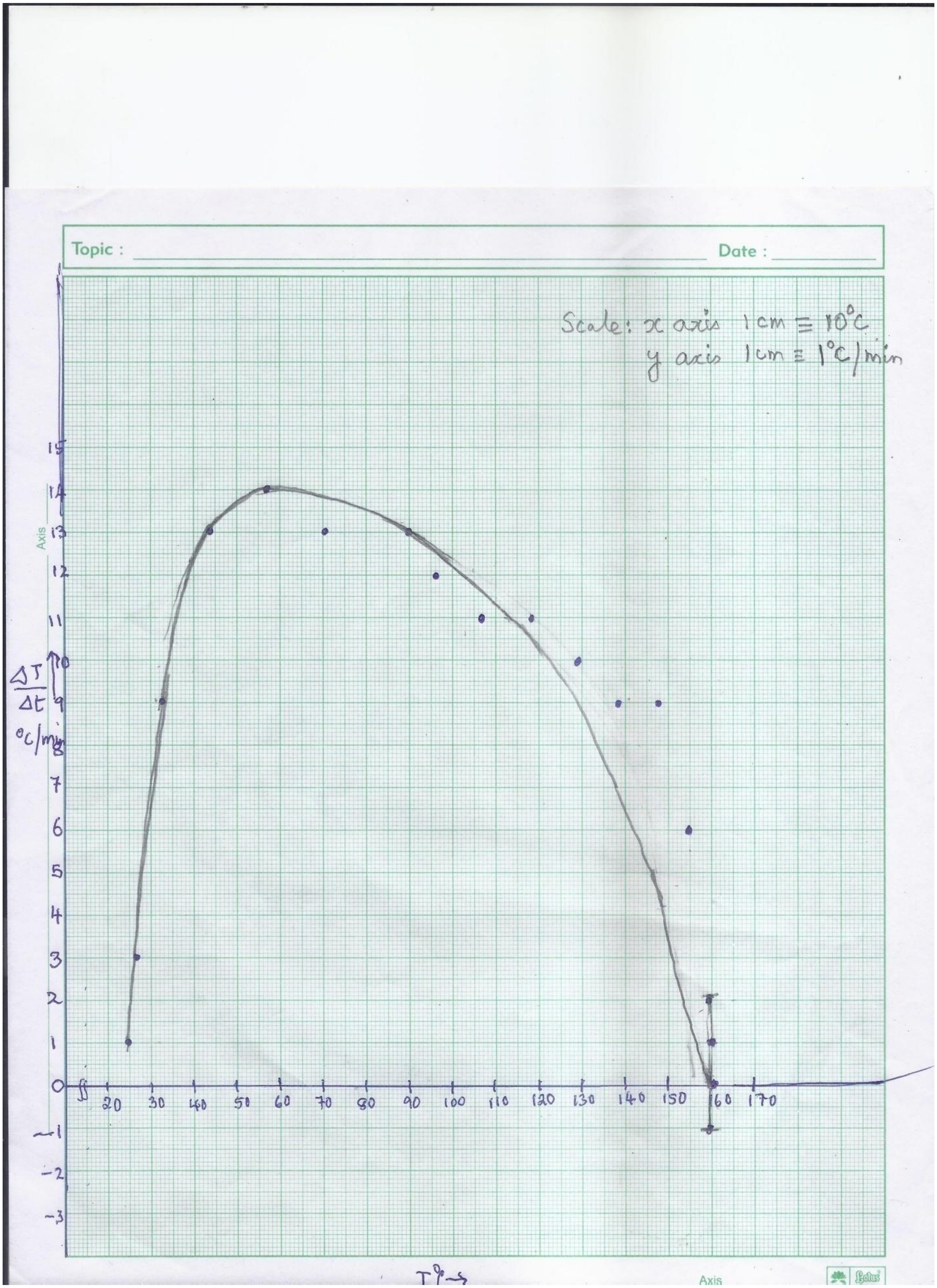
(Make an appropriate Table for the plot)

t(min)	T= (T1+T2)/2^{0}C	ΔT =T2-T1^{0}C	Δt = t2-t1(min)	ΔT/Δt ^{0}Cmin^{-1}
0				
1	24.5	1	1	1
2	26.5	3	1	3
3	32.5	9	1	9
4	43.5	13	1	13
5	57	14	1	14
6	70.5	13	1	13
7	83.5	13	1	13
8	96	12	1	12
9	107.5	11	1	11
10	118.5	11	1	11
11	138.5	10	1	10
12	147.5	9	1	9
13	155	9	1	9
14	159	6	1	6

15	160	2	1	2
16	160	0	1	0
17	160	0	1	0
18	159.5	-1	1	-1
19	159.5	1	1	1
20	160	0	1	0
21	160.5	1	1	1
22	160.5	-1	1	-1
23	160.5	1	1	1

(marking scheme: 1 mark for making an appropriate table and choosing T to be an average;0.5 marks for correct ΔT; 0.5 marks for using the whole graph paper; 0.5 marks for indicating scale on x axis; 0.5 marks for indicating scale on y axis; 1.5 marks for a smooth plot; 0.5 marks for indicating fluctuations around 160^0C)

Roll No. [ ][ ][ ][ ] - [ ][ ][ ][ ] - [ ][ ][ ][ ]



Topic :
Date :
Scale: x axis 1cm ≡ 10°C
y axis 1cm ≡ 1°C/min
ΔT/Δt °c/min
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 −1 −2 −3
20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170
T°→
Axis

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b)(i) Identify the temperature T at which dT/dt becomes zero. (1)

From the graph at around 160^0C, neglecting random fluctuation, the Temp. is constant.

T = 160^0C

(ii) From(i) which intrinsic property of the liquid can be inferred ? What is the value? (2 + 1)

Intrinsic property: Boiling point

Value:160^0C

(iii) Give two possible reasons as to why the there is a 1^0 fluctuation in temperature after 15 mins. (2)

Least count of instrument is 1^0C; Statistical fluctuations such as non-uniform heating by the hot plate, convective corrections, radiation etc.

c) Plot m vs. t after t= 15 min. (4)

(Marking scheme: 0.5 for using the full graph paper; 0.5 for mentioning scaling of x axis; 0.5 for mentioning scaling of y axis; 2.0 for drawing a smooth graph and taking an average line; 0.5 points for choosing two far off points to calculate slope)

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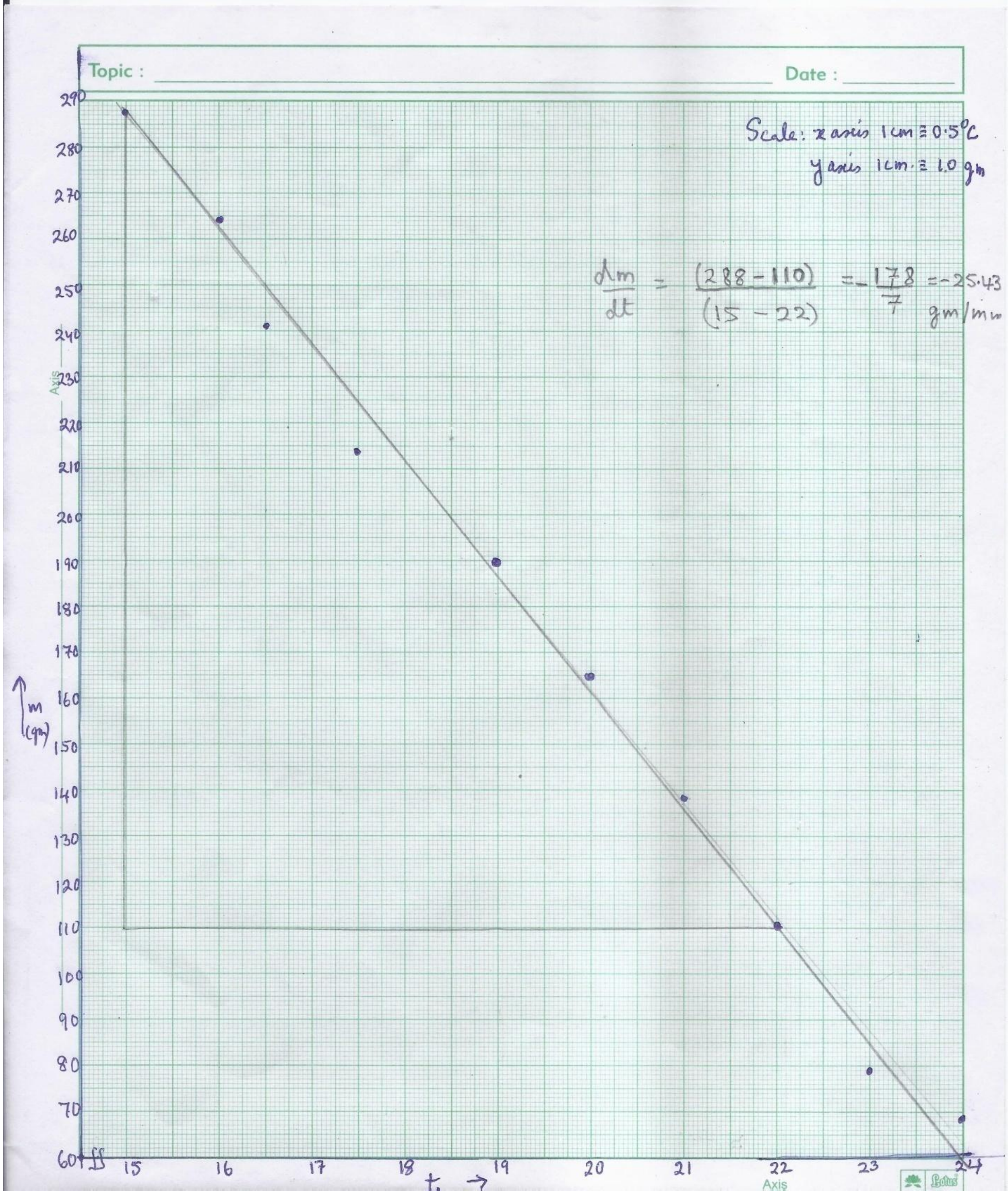

Topic :
Date :
Scale: x axis 1cm ≡ 0.5°C
y axis 1cm ≡ 1.0 gm
dm/dt = (288 − 110)/(15 − 22) = −178/7 = −25.43 gm/min
m (gm)
t →
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
15
16
17
18
19
20
21
22
23
24
Axis

d) It is established from other experiments that at t=19min, the liquid z absorbs heat from the hot plate at the rate 84.0W. Using this information what intrinsic property of the liquid, other than its boiling point, may be inferred? Find its value (2 + 3)

Latent heat L (2)

$(dQ/dt)_{t=22min} = L\,(dm/dt)_{t=22min}$ (1)

From graph $(dm/dt)_{t=22min}$ =25.4gm/min = 0.42×10^{-3}Kg/sec (1)

$84 = L \times 0.4238 \times 10^{-3}$

$L = 84/0.4238 \times 10^{-3} = 198$ kJ Kg^{-1} (1 for value correct significant figures consistent with data)

Intrinsic property: Latent heat   Value: 198kJ Kg^{-1} (The accuracy of instruments)

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Q6) A Schwarzschild black hole is characterized by its mass M and a mathematical spherical surface of radius $R_S = \frac{2GM}{c^2}$ called the event horizon. If the radial distance of an object r from the black hole is such that $r < R_s$, then the object is "swallowed" by the black hole and r rapidly decreases to the singular point $r = 0$.

a) Suppose a black hole of mass M "captures" a proton to form a "black hole proton atom (BHP)". Find the smallest radius r_B of this atom. (3)

If m is the mass of the proton and v the speed in a circular orbit, then with $m <<< M$ $m\frac{v^2}{r} = \frac{GmM}{r^2}$ (0.5)

Bohr's quantization condition gives $mvr = n\hbar$ where r is the radius of the orbit (0.5)

Hence $v = \sqrt{(\frac{GM}{r})}$ Since $v = \frac{n\hbar}{mr}$ (0.5)

$r = \frac{n^2\hbar^2}{GM\,m^2}$ Hence the smallest radius $r_B = \frac{\hbar^2}{GM\,m^2}$ (0.5 +1)

$$r_B = \frac{\hbar^2}{GM\,m^2}$$

b) Obtain a numerical upper bound on M such that a stable BHP may exist. (2)

For a stable BHP to exist $r_B > R_s$ (0.5)

i.e. $\frac{\hbar^2}{GM\,m^2} > \frac{2GM}{c^2}$ Thus $M^2 < \frac{(\hbar c)^2}{2(Gm)^2}$ or $M < \frac{(\hbar c)}{\sqrt{2}(Gm)}$ (1.0)

$M < 2.$ X 10^{11}Kg. (Correct decimal place consistent with values given 0.5)

$M < 2.X\ 10^{11}$Kg.

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c) Find the minimum energy E_{min} , in Mev ,required to dissociate this BHP atom from the ground state. . (2)

The energy of the BHP = $E = -0.5\frac{GMm}{r}$, Substituting for r, $E_n = -0.5\frac{G^2M^2m^3}{n^2\hbar^2}$ (1.0)

Thus the min energy required to dissociate the atom is $E_{min} = 0.5\frac{G^2M^2m^3}{\hbar^2}$ (0.5)

Choose M = Kg , substituting values $E_{min} = 55$ MeV (0.5)

$E_{min} = 55$ MeV

d) In 1974, Stephen Hawking showed that quantum effects cause black holes to radiate like a black body with temperature $T_{BH} = \frac{10^{23}K}{M}$. Discuss then the possibility of the existence of a stable BHP atom. (3)

For $M = 10^{11}$ Kg. $T_{BH} = \frac{10^{23}K}{10^{11}} = 10^{12}K^0$ At this temperature thermal energies $\simeq kT_{BH} = 82\ MeV$ The dissociation energy required is 55 MeV. Thus the BHP is thermally unstable.