

1. Prove that there do not exist natural numbers  $x$  and  $y$ , with  $x > 1$ , such that

$$\frac{x^7 - 1}{x - 1} = y^5 + 1.$$

2. In a triangle  $ABC$ ,  $AD$  is the altitude from  $A$ , and  $H$  is the orthocentre. Let  $K$  be the centre of the circle passing through  $D$  and tangent to  $BH$  at  $H$ . Prove that the line  $DK$  bisects  $AC$ .
3. Consider the expression

$$2013^2 + 2014^2 + 2015^2 + \cdots + n^2.$$

Prove that there exists a natural number  $n > 2013$  for which one can change a suitable number of plus signs to minus signs in the above expression to make the resulting expression equal 9999.

4. Let  $ABC$  be a triangle with  $\angle A = 90^\circ$  and  $AB = AC$ . Let  $D$  and  $E$  be points on the segment  $BC$  such that  $BD : DE : EC = 1 : 2 : \sqrt{3}$ . Prove that  $\angle DAE = 45^\circ$ .
5. Let  $n \geq 3$  be a natural number and let  $P$  be a polygon with  $n$  sides. Let  $a_1, a_2, \dots, a_n$  be the lengths of the sides of  $P$  and let  $p$  be its perimeter. Prove that

$$\frac{a_1}{p - a_1} + \frac{a_2}{p - a_2} + \cdots + \frac{a_n}{p - a_n} < 2.$$

6. For a natural number  $n$ , let  $T(n)$  denote the number of ways we can place  $n$  objects of weights  $1, 2, \dots, n$  on a balance such that the sum of the weights in each pan is the same. Prove that  $T(100) > T(99)$ .

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