1. Let ABC be an acute-angled triangle. The circle Γ with BC as diameter intersects AB and AC again at P and Q, respectively. Determine $\angle BAC$ given that the orthocentre of triangle APQ lies on Γ.

2. Let $f(x) = x^3 + ax^2 + bx + c$ and $g(x) = x^3 + bx^2 + cx + a$, where a, b, c are integers with $c \neq 0$. Suppose that the following conditions hold:

   (a) $f(1) = 0$;

   (b) the roots of $g(x) = 0$ are the squares of the roots of $f(x) = 0$.

   Find the value of $a^{2013} + b^{2013} + c^{2013}$.

3. Find all primes p and q such that p divides $q^2 - 4$ and q divides $p^2 - 1$.

4. Find the number of 10-tuples $(a_1, a_2, \ldots, a_{10})$ of integers such that $|a_1| \leq 1$ and

$$a_1^2 + a_2^2 + a_3^2 + \cdots + a_{10}^2 - a_1a_2 - a_2a_3 - a_3a_4 - \cdots - a_9a_{10} - a_{10}a_1 = 2\,.$$

5. Let ABC be a triangle with $\angle A = 90°$ and $AB = AC$. Let D and E be points on the segment BC such that $BD : DE : EC = 3 : 5 : 4$. Prove that $\angle DAE = 45°$.

6. Suppose that m and n are integers such that both the quadratic equations $x^2 + mx - n = 0$ and $x^2 - mx + n = 0$ have integer roots. Prove that n is divisible by 6.

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